Rare event analysis using the Limited Relative Error Algorithm for OMNeT++ simulations

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Motivation
Stochastic simulation $\rightarrow$ *statistical evaluation* $\rightarrow$ objective statement

- IID Property
- Batch Means
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- Confidence Interval
Stochastic simulation → statistical evaluation → objective statement

Evaluation methods using Confidence Intervals

1. **Batch Means**
   
   1 simulation run → \((x_1, x_2, \ldots, x_n)\) observations
   
   → split into \(k\) batches of \(b\) observations (\(n = kb\))
   
   → find batch means \(Y_i(b)\)
   
   → reduce sample correlation by forming “quasi-independent, quasi-normally distributed batch-random variables”
   
   “deficient” according to [1]
   
   → what's the right batch length and simulation time?
Stochastic simulation $\rightarrow$ statistical evaluation $\rightarrow$ objective statement

Evaluation methods using Confidence Intervals

2. Replication method
   $i$ simulation runs $\rightarrow [(x_{1,1}, x_{2,1}, \ldots, x_{n,1}), \ldots, (x_{1,i}, x_{2,i}, \ldots, x_{n,i})]$  
   $\rightarrow i$ mean values, one per repetition  
   $\rightarrow$ repetition of same scenario eliminates correlation  
   $\rightarrow$ have to eliminate warmup period  
   $\rightarrow$ runs need to be long enough to be iid
Motivation, cont.

- How do you know *a-priori*
  - how many observations or repetitions are required
  - what the simulation time should be

for a *statistically sound* analysis?

⇒ Akaroa2, from [2]:
  - runs distributed simulations
  - merges results centrally
  - analyses results online
  - stops processes once results are deemed confident enough

- Confidence intervals break for very rare and very likely events
Limited Relative Error (LRE) attempts to

(a) approximate an unknown cumulative distribution function (CDF) function $F_X(x)$ as $\tilde{F}_X(x)$,
(b) make statements about the sample sequence correlations,
(c) determine a relative error function,
(d) request more samples until an error bound is met,
(e) requires a single simulation run and monitors sample correlation,
(f) is designed to work well with very rare events.
Confidence interval-based methods evaluate the mean of a statistic
⇒ suited to obtain a picture of the range of a statistic
fails for very rare / likely events (Normal distribution
assumption doesn’t hold)
“What is the average packet delay this system achieves?”

LRE evaluates the distribution of a statistic
⇒ suited for reliability analysis
can specify target resolution and max. accepted error
beforehand
“How likely is it that this system experiences VoIP packet delays > 150 ms”
Algorithm Description
Markov chains in LRE

Figure 1: Graphical visualization of $G(x)$ in Equation 1.

1. Obtain observations $(x_1, x_2, \ldots, x_n)$
2. $(x_1, x_2, \ldots, x_n)$ corresponds to $(k + 1)$-state Markov chain
3. For this, the complementary cumulative distribution function (CCDF) $G(x)$ can be found as

$$G(x) = G_i = P(X > x) = \sum_{j=i}^{k} P_j \text{ for } i - 1 \leq x < i, \ i = 1, 2, \ldots, k$$

with $G_0 = 1$ and $G_{k+1} = 0$
The local correlation coefficient can be found as

\[ \rho(x) = 1 - (p_0(x) + p_1(x)) \]  

(2)
1. Goal: determine the CCDF $G(x)$ where transition probabilities $p_{ij}$ are initially *not* known.
2. Count how many times each state has been entered in counter $h_i$ after $n$ transitions.
3. Find right state $S_1(x)$ frequency $v_i = \sum_{j=i}^{k} h_j$ for $i = 0, 1, \ldots, k$, $v_0 = n$
left state $S_0(x)$ frequency $r_i = n - v_i$. 

\[ v_i = \sum_{j=i}^{k} h_j \]
4. Count $S_1(x) \rightarrow S_0(x)$ transition frequency in $c_i$ ($a_i$ in an analogue way).
5. We can now find $\tilde{G}(x) = \frac{v_i}{n}$, $\tilde{\rho}(x)$ and the relative error $d_i = \sigma_{G(x)}/\tilde{G}(x)$.
The relative error $d_i$

$$d_i = \frac{\sigma_{G(x)}}{\hat{G}(x)} \cong \text{confidence of results}$$

- $\sigma_{G(x)}$ is the (normally distributed [4] [5]) standard deviation of $S_1(x)$ in the 2-state Markov chain and a function of the correlation factor $\tilde{cf}(x) = \frac{1+\tilde{\rho}(x)}{1-\tilde{\rho}(x)}$

- $d_i = \frac{\text{absolute error}}{P(\text{in state } \geq i)} \Rightarrow \text{relative error}$

- many transitions from a state to itself $\Rightarrow$ large $\tilde{\rho}(x) \Rightarrow$ large $d_i$

$\Rightarrow$ algorithm demands more observations to ensure accurate modeling of transitions between states
Simulation ends when \( d_i \leq d_{\text{max}} \) for all \( i = 0, \ldots, k \).
Usage
The LRE algorithm as a standalone version is available as open-source at [6] (based on openWNS simulator [7]).

The OMNeT++ integration is available as open-source at [8].

A novel LRE entity can be added to network models.

- It is easily configured in the .ini file.

```ini
**.lre.xmin = 0.0
**.lre.xmax = 1.0
**.lre.bin_size = 0.1
**.lre.max_error = $\{e=0.01..0.05\ step 0.005\} \ # \ usually \ just \ 1 \ value
**.lre.evaluation_interval = 1000
**.lre.output_file = "lre_output_evaluation_e${e}.txt"
```
Example results

(a) Number of observations LRE requested (b) CCDF LRE computed for different $d_{\text{max}}$. for different $d_{\text{max}}$. 
LRE is an alternative method to determine the confidence of simulation results.

LRE determines when a simulation should end to obtain confident results in the desired range.

The intended resolution of the statistic must be input a-priori.

⇒ It is suited for reliability analysis, where known performance bounds can be tested.

It is convenient to use: easy configuration, single run, no post-processing.

A new algorithm description has been given.

The algorithm is made available as open-source both standalone and as an OMNeT++ integration.

The combination with the RESTART method ([9] [10]) could reduce simulation time for very rare events could prove very useful to researchers.
Thank you very much for your attention! :)}


